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On the mean values of an entire function in several complex variables represented by multiple Dirichlet series

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Abstract
This paper intends to study the properties of mean values of an Entire Function Represented by Multiple Dirichlet Series. Concepts of mean values of an entire function represented by Dirichlet series in one complex variable are extended to an entire function of several complex variables represented by Multiple Dirichlet Series.

Keywords: Entire function, several complex variables, multiple Dirichlet series

Introduction
The properties of mean values of an entire function represented by Dirichlet series in one complex variable have been studied by various authors to a considerable extent.
Our purpose here is to extend these concepts of mean values of an entire function represented by Dirichlet series in one complex variable to an entire function of several complex variables represented by Multiple Dirichlet Series. where for the sake of simplicity, case of two variables is considered instead of several variables.

Let us consider

(1.1) $f(s_1, s_2) = \sum_{m,n=1}^{\infty} a_{m,n} \exp(s_1\lambda_m + s_2\mu_n) \; \left((s_j = \sigma_j + it_j), j = 1,2\right)$

where $a_{m,n} \in C$, the field of complex numbers, $\lambda'_m s, \mu'_n s$ are real, and

$0 < \lambda_1 < \lambda_2 < \cdots \lambda_m \to \infty;$

$0 < \mu_1 < \mu_2 < \cdots < \mu_n \to \infty.$

A.I. Janusauskas in his paper (Janusauskas 1977) has shown that if

(1.2) $\lim_{m\to\infty} \frac{\log m}{\lambda_m} = 0, \lim_{n\to\infty} \frac{\log n}{\mu_n} = 0,$

then the domain of convergence of the series (1.1) coincides with its domain of absolute convergence.

The necessary and sufficient condition that the series (1.1) satisfying (1.2) to be entire shown by Sarkar [2, pp.99] is that

(1.3) $\lim_{(m,n)\to\infty} \frac{\log|a_{m,n}|}{\lambda_m + \mu_n} = \infty$

Throughout F stands for all double Dirichlet series of the form (1.1) satisfying (1.2) and (1.3) Then $f \in F$ denotes an entire function over C^2, the cartisian product of two copies of the complex plane. The results can be extended to several complex variables.

Sarkar [1, pp100] has defined that Corresponding to an f$\in F$, the maximum modulus M =M_f and the maximum term $\mu = \mu_f$ on $R^2$are defined as

$$M(\sigma) = M_f(\sigma_1, \sigma_2) = \max\{|f(s_1, s_2)|: s_1, s_2 \in C, \mathrm{Re}\, s_1 = \sigma_1, \mathrm{Re}\, s_2 = \sigma_2\}$$

$$\mu(\sigma) = \mu_f(\sigma_1, \sigma_2) = \max_{(m,n)\in N^2} \{|a_{m,n}| \exp(\sigma_1\lambda_m + \sigma_2\mu_n)\}$$

$w$here N is the set of natural numbers.

We define the mean values of $|f(s_1, s_2)|$ is defined as

$$(1.4)\ I_2(\sigma_1, \sigma_2\,; f) = I_2(\sigma_1, \sigma_2) = \lim_{T_1 T_2 \to \infty} \frac{1}{4T_1T_2} \int_{-T_1}^{T_1} \int_{-T_2}^{T_2} |f(\sigma_1 + it_1, \sigma_2 + it_2)|^2 dt_1\, dt_2$$

And mean value $m_{2,k}(\sigma_1, \sigma_2)$ of $|f(s_1, s_2)|$ as

$$(1.5)\ m_{2,k}(\sigma_1, \sigma_2; f) = m_{2,k}(\sigma_1, \sigma_2)$$

$$= \lim_{T_1, T_2 \to \infty} \frac{1}{T_1 T_2 e^{k\sigma_1} e^{k\sigma_2}} \int_0^{\sigma_1} \int_0^{\sigma_2} \int_{-T_1}^{T_1} \int_{-T_2}^{T_2} \{|f(x_1 + it_1, x_2 + jt_2)|^2 e^{kx_1} e^{kx_2}\} dx_1 dx_2 dt_1 dt_2$$

where k is any positive number.
From (1.4) and (1.5), we can write

$$(1.6)\ m_{2,k}(\sigma_1, \sigma_2) = \frac{4}{e^{k\sigma_1} e^{k\sigma_2}} \int_0^{\sigma_1} \int_0^{\sigma_2} I_2(x_1, x_2) e^{kx_1} e^{kx_2} dx_1\, dx_2$$

2. Theorem 1: For the Dirichlet series $f(s_1, s_2)$, f $\epsilon\ F, I_2(\sigma_1, \sigma_2)$ is an increasing function of σ_1 *and* σ_2.

Proof: We have

$$|f(s_1, s_2)|^2 = f(s_1, s_2)\overline{f(s_1, s_2)}$$

$$= \sum_{m_1 n=1}^{\infty} |a_{m,n}|^2 \exp\{2(\sigma_1 \lambda m + \sigma_2 \mu_n)\}$$

$$+ \sum_{m \neq M} \sum_{n \neq N} a_{m,n} \bar{a}_{M,N} \exp\{\sigma_1(\lambda_m + \lambda_M) + \sigma_2(\mu_n + \mu_N) + it_1(\lambda_m - \lambda_M) + it_2(\mu_n - \mu_N)\}$$

Since both the series on the right are absolutely convergent, the resulting series is uniformly convergent for any finite t_1 and t_2 range, therefore we may integrate term by term for finite t_1 and t_2. Hence on integration all the terms for which m$\neq M$, n$\neq N$, vanish as $T_1, T_2 \to \infty$ and we obtain

$$(2.1)\ I_2(\sigma_1, \sigma_2) = \sum_{m,n=1}^{\infty} |a_{m,n}|^2 \exp\{2(\sigma_1 \lambda_m + \sigma_2 \mu_n)\}$$

It is clear from the value of $I_2(\sigma_1, \sigma_2)$ that it is an increasing function of σ_2 for a fixed value of σ_1 and vice versa. Hence $I_2(\sigma_1, \sigma_2)$ is an increasing function of both σ_1 and σ_2.

Corollary 1: For the Dirichlet series $f(s_1, s_2)$, f $\epsilon\ F$,

$$\{\mu(\sigma_1, \sigma_2)\}^2 \leq I_2(\sigma_1, \sigma_2) \leq \{M(\sigma_1, \sigma_2)\}^2$$

This follows from the definitions of M(σ_1, σ_2), μ(σ_1, σ_2) and (2.1).

3. Theorem 2: For the Dirichlet series $f(s_1, s_2)$, f $\epsilon\ F$, $m_{2,k}(\sigma_1, \sigma_2)$ is an increasing function of σ_1 *and* σ_2.
Proof: We have from (1.6)

$$m_{2,k}(\sigma_1, \sigma_2) = \frac{4}{e^{k\sigma_1} e^{k\sigma_2}} \int_0^{\sigma_1} \int_0^{\sigma_2} I_2(x_1, x_2) e^{kx_1} e^{kx_2} dx_1\, dx_2$$

Using (2.1), we obtain

$$m_{2,k}(\sigma_1,\sigma_2) = \frac{4}{e^{k\sigma_1}e^{k\sigma_2}} \int_0^{\sigma_1} \int_0^{\sigma_2} [\sum_{m,n=1}^{\infty} |a_{m,n}|^2 \exp\{2(\sigma_1\lambda_m + \sigma_2\mu_n)\} e^{kx_1} e^{kx_2} dx_1\, dx_2]$$

$$= \frac{4}{e^{k\sigma_1}e^{k\sigma_2}} \sum_{m,n=1}^{\infty} |a_{m,n}|^2 \int_0^{\sigma_1} \int_0^{\sigma_2} e^{(2\lambda_m + k)x_1}\, e^{(2\mu_n + k)x_2}\, dx_1\, dx_2]$$

$$= \frac{4}{e^{k\sigma_1}e^{k\sigma_2}} \sum_{m,n=1}^{\infty} [\frac{|a_{m,n}|^2 \left(e^{(2\lambda_m + k)\sigma_1} - 1\right)\left(e^{(2\mu_n + k)\sigma_2} - 1\right)}{(2\lambda_m + k)(2\mu_n + k)}]$$

(3.1) $m_{2,k}(\sigma_1,\sigma_2) = 4 \sum_{m,n=1}^{\infty} \frac{|a_{m,n}|^2 \left(e^{2\lambda_m\sigma_1} - e^{-k\sigma_1}\right)\left(e^{2\mu_n\sigma_2} - e^{-k\sigma_2}\right)}{(2\lambda_m + k)(2\mu_n + k)}$

Thus $m_{2,k}(\sigma_1,\sigma_2)$ that it is an increasing function of σ_2 for a fixed value of σ_1 and vice versa. Hence $m_{2,k}(\sigma_1,\sigma_2)$ is an increasing function of both σ_1 and σ_2.

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